Induction and Rigid Body Dynamics

To what extent can we determine the behaviour of the magnetic torque with respect to time produced on a conducting and rotating rectangular loop from a varying magnetic flux?

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Contents

1 Introduction

I grew up obsessed with Lego. I would love to build cars, robots, planes and anything my mind could imagine. As a physics student I now understand that machinery is not as simple as putting blocks together. Machinery takes power and electricity. This is why I chose to investigate the phenomenon that powers our life: electromagnetic induction. Both electric motors and electric generators depend on this principle to function. Through its application, motors turn electrical energy into mechanical energy and generators turn mechanical energy into electrical energy. Although both machines function on the same principle they are very different from one another. For example, an induction motor depends on the torque generated from the induced current to operate, whilst the generator can either benefit or suffer from the torque production. For the present investigation, I will analyze a system where a torque is generated from an induced current, a system that assimilates aspects of both electric machines. Consisting of a rotating loop in the presence of a magnetic field, like in a generator, but the magnetic field is going to rotate, like in a motor. The objective of this investigation is to model the behaviour of the torque with respect to time and analyze its implications on the rotation of the loop. Thus, we look to answer the question: to what extent can we determine the behaviour of the magnetic torque with respect to time produced on a conducting and rotating rectangular loop from a varying magnetic flux? This will be achieved through a theoretical and mathematical analysis of the system and verified with the data of a simulator.

2 Theoretical framework

2.1 Magnetic Field

A magnetic field ($\vec{B}$) is a vector field that depicts the flow of the flux lines produced from the influence of a current or a magnet. For a current, according to its direction we apply the right hand rule to determine the orientation of its magnetic field. For a permanent magnet the flow of the field will always be the same, diverging from the north pole and converging into the south pole.

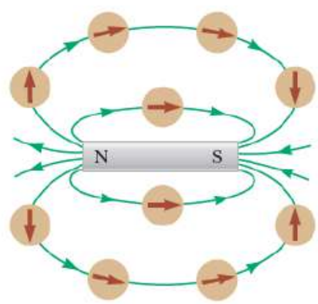

Figure 1: Magnetic field of a permanent magnet. [1]

In 1882, by varying the currents passing through 4 electromagnets, Nikola Tesla developed an essential component of the modern AC induction motor: the rotating magnetic field (R.M.F)[1][2]. For the present investigation, we will simulate this phenomenon by rotating a permanent magnet causing the field to move accordingly.

2.2 Magnetic Flux

The magnetic flux (ϕ_B) is a measure of how many magnetic field lines pass through a surface. For any surface it is expressed as:

$$\phi_B = \int \vec{B} \cdot d\vec{A} \tag{1}$$

Where $\vec{B} \cdot d\vec{A}$ is the scalar product of the magnetic field and $d\vec{A}$, a vector normal to the infinitesimal

[1]See more about the R.M.F of an AC induction motor in Appendix A

surface area. We can express this according to their magnitudes and unit vectors:

$$\phi_B = \int (B\hat{B}) \cdot (dA\hat{n})$$

Where $\hat{B}$ is the unit vector that represents the direction of the magnetic field and $\hat{n}$ is the unit vector normal to the infinitesimal surface area.

$$\phi_B = \int BdA(\hat{B} \cdot \hat{n})$$

However, if the intensity of the magnetic field is constant throughout every part of the surface, then, B comes out of the integral and using the definition of vector scalar product, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$ [3], we express the flux as:

$$\hat{B} \cdot \hat{n} = |1||1| \cos\theta = \cos\theta$$

$$\phi_B = B\cos\theta \int dA$$

Considering the integral of an area is the area. We reach the expression for magnetic flux when the magnetic field does not depend on the area:

$$\phi_B = BA\cos\theta \tag{2}$$

Where θ is the angle formed between the normal of the surface and the magnetic field lines.

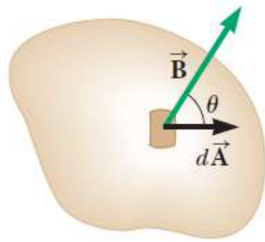

Figure 2: The figure shows the magnetic field making an angle θ with the vector normal to the infinitesimal surface area. [1]

2.3 Faraday's Law of Induction

In 1831, Michael Faraday realized 3 experiments making use of a loop of wire and a magnet. He concluded that the relative motion of the magnetic field with respect to the loop induces a current [4]. This phenomenon is known as electromagnetic induction, Faraday was the first scientist to make note of its existence and express it into a mathematical expression known as Faraday's law:

$$\varepsilon = -\frac{d\phi_B}{dt} \tag{3}$$

His law states that the induced emf is proportional to the rate of change of magnetic flux with respect to time. The emf generates an electric field inside the conductor that will cause current to flow. So, the faster the magnetic field changes with time, the more current will be induced in the loop. The direction of the induced current as well as the negative sign of the equation are explained through Lenz's law.

Nowadays, Faraday's law is better interpreted through Maxwell's work [5]. James Clerk Maxwell, after Faraday, from a geometric point of view, found that a varying magnetic flux with respect to time over the area enclosed by the conductor, produces an electric field, that follows a closed path, tangent to the conductor:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

Where the left hand side of the equation represents the closed path followed by the electric field in the conductor, the right hand side, represents the changing magnetic flux with respect to time. This is known as the Maxwell-Faraday law in integral form.

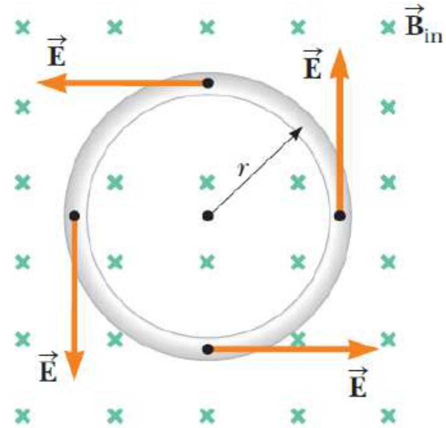

Figure 3: An electric field is generated by a change in magnetic flux with respect to time, over the area enclosed by the conductor. The generated electric field follows a closed path, tangent to the conductor. [1]

2.4 Lenz's Law

Lenz's law states that the direction of the induced current in a loop will be orientated as to generate a magnetic flux that compensates the change in magnetic flux. In other words, the induced current will flow in the direction that generates a magnetic field that opposes the original one. This way conserving the original magnetic flux through the loop.

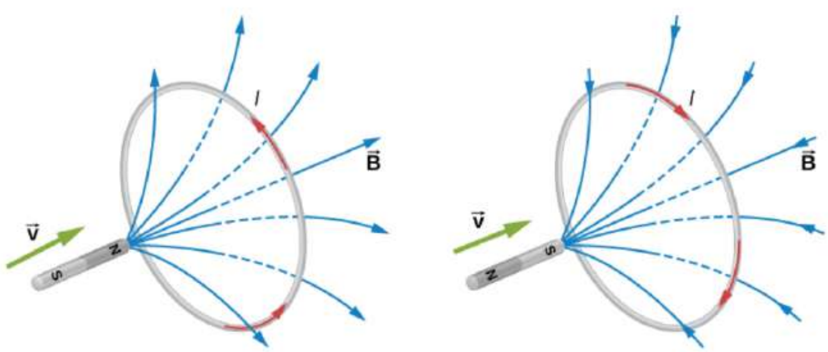

Figure 4: Moving magnet induces a current whose magnetic field opposes the increment of the original one. [6]

2.5 Magnetic Force in a Conductor

A charged particle moving through a magnetic field experiences a magnetic force

$$\vec{F_B} = q\vec{v} \times \vec{B}$$

Where $\times$ is the vector product operator. For a current carrying conductor, the equation is extended to every particle moving inside. Henceforth, we multiply it by the number of charges and the volume of the conductor.

$$\vec{F_B} = (q\vec{v} \times \vec{B})nAL$$

Replacing the value of current $I = nqvA$ we reach a final equation for the magnetic force in a current carrying conductor.

$$\vec{F_B} = I\vec{L} \times \vec{B}$$

Where $\vec{L}$ points in the direction of the current. Using the vector product definition, $|\vec{L} \times \vec{B}| = |\vec{L}||\vec{B}| \sin\theta$[3], where θ, is the angle formed between the vector $\vec{L}$ and the magnetic field. Therefore, the magnitude of the magnetic force on a current is:

$$F_B = ILB \sin\theta \tag{4}$$

It is important to mention that the force in the conductor will always be orthogonal to the direction of the current and magnetic field.

2.6 Torque

Like the cause of change in translational motion is the force, the cause of change in rotational motion is the torque (τ). This is defined as the tendency of a force to rotate a body about an axis [7], generating either a clockwise or anticlockwise rotation. Mathematically, torque is defined as the vector product of vector $\vec{r}$ (a position vector which points to the point of force application from the rotation axis) and $\vec{F}$ (the force applied at some point over the body).

$$\tau = \vec{r} \times \vec{F}$$

Using the vector product definition, $|\vec{r} \times \vec{F}| = |\vec{r}||\vec{F}| \sin\theta$, where θ, is the angle formed between the vector $\vec{r}$ and $\vec{F}$. We write the magnitude of the torque as:

$$\tau = Fr \sin\theta \tag{5}$$

Measured in $N.m$, where $r \sin\theta$ is the lever arm, marking the perpendicular distance from the axis of rotation to the point where the force is applied. Although, torque is a vector quantity, for the present investigation we are only interested in its magnitude and how it should affect the loop. Therefore, it should be noted that for rotating object, a negative torque would oppose its rotation but a positive torque would complement it.

3 Theoretical Development

3.1 System to be studied

The system we will be analyzing consists of a permanent magnet next to a rectangular loop. The magnet is to the left of the loop and has an axis of rotation at its center, perpendicular to the page. The rectangular loop with length a and width b has an axis of rotation passing through its middle directed in the vertical axis. Initially, the magnet's north pole is parallel to the plane: $\phi_B = 0$ as the field is perpendicular with the normal of the area enclosed. However, a current will be induced and a torque will be generated as both bodies rotate.

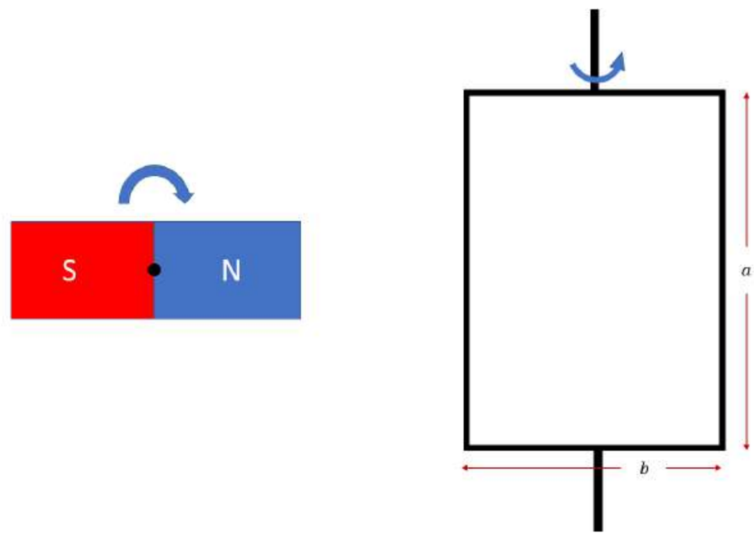

Figure 5: The magnet rotates clockwise and the loop anti-clockwise, each with their respective angular velocity

3.1.1 Induced Current

For this general case, both the magnetic field and area rotate with their respective angular velocity. We describe the variation of their magnitudes through the use of trigonometric functions. The magnetic field is assigned a cosine function because it starts at a maximum.

$$B(t) = B_o \cos(\omega_B t)$$

On the contrary, the magnetic flux over the area enclosed by the loop is initially 0. Therefore, a sine function is appropriate for its description:

$$A(t) = ab\sin(\omega_C t)$$

Where ω_C is the angular velocity of the conductive loop. Henceforth, the magnetic field does not depend on the area enclosed by the loop, but instead varies with respect to time. This way we use equation 2 and replace the functions:

$$\phi_B(t) = B(t)A(t)$$

Where $\cos\theta$ is absorbed by the trigonometric terms:

$$\phi_B(t) = B_o\cos(\omega_B t)(ab)\sin(\omega_C t)$$

To facilitate our calculation we'll move all the constants to one side:

$$\phi_B(t) = B_o(ab)[\cos(\omega_B t)\sin(\omega_C t)]$$

We take the derivative of the magnetic flux with respect to time. Employing the "product rule" to the terms in between the brackets.

$$\frac{d\phi_B}{dt} = B_o ab[-\omega_B\sin(\omega_B t)\sin(\omega_C t) + \omega_C\cos(\omega_C t)\cos(\omega_B t)]$$

We multiply $\frac{d\phi}{dt}$ by -1 to reach an expression for the induced emf:

$$\varepsilon(t) = -\frac{d\phi_B}{dt} = B_o ab[\omega_B \sin(\omega_B t)\sin(\omega_C t) - \omega_C \cos(\omega_C t)\cos(\omega_B t)]$$

Finally, assuming the loop is made of an ohmic material, we use Ohm's law to reach an expression for the induced current, from a rotating magnetic field:

$$\varepsilon(t) = I(t)R$$

$$I(t) = \frac{B_o ab[\omega_B \sin(\omega_B t)\sin(\omega_C t) - \omega_C \cos(\omega_C t)\cos(\omega_B t)]}{R} \tag{6}$$

3.1.2 Torque produced from EMF

From Lenz's law, we can deduce that the induced current will flow clockwise when the north pole faces the area of the loop and counter clockwise when the south pole faces the area. This being said, the magnetic force will be the same for either instance as the direction of current and field vary simultaneously. Therefore, an analysis of either scenario suffices.

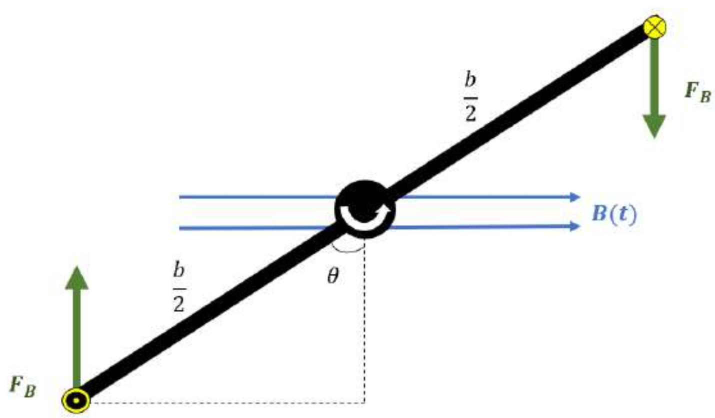

Figure 6: Birds eye view of the system when the induced current flows counterclockwise through the loop

Figure 6, shows the direction of the magnetic forces experienced by the current flowing vertically through the lengths of the loop. Since only the magnetic field perpendicular to the flow of current generates the magnetic forces, we can express their magnitude as such:

$$F_B = IaB\sin\left(\frac{\pi}{2}\right)$$

$$F_B = IaB$$

Where $I = I(t)$ and $B = B(t)$ and the magnetic force is directed orthogonal to the vertical flow of current and the horizontal field lines, as depicted in figure 6. We will not consider the magnetic forces experienced by the current flowing through the width as they propose no effect over the rotation of the loop.

To calculate the torque generated by each force first we have to deduce the lever arm. If the width is "b", then the distance from the axis of rotation to the point of force is $\frac{b}{2}$. Likewise, according to figure 6, the angle θ determines the perpendicular component of the distance. Therefore, the lever arm is:

$$\frac{b}{2}\sin\theta$$

Consequently, the magnitude of torque for a single side is expressed as:

$$\tau = F_B\frac{b}{2}\sin\theta$$

To determine the net torque we add both torques in their complete expressions.

$$\tau_{net} = IaB\frac{b}{2}\sin\theta + IaB\frac{b}{2}\sin\theta = IabB\sin\theta$$

We can express θ in terms of ω_C considering that initially $\theta = \frac{\pi}{2}$ but its value decreases according to the angular velocity of the loop.

$$\tau_{net} = IabB\sin\left(\frac{\pi}{2} - \omega_C t\right)$$

$$\tau_{net} = IabB\cos(\omega_C t)$$

To reach a complete expression for the torque, we'll replace the predetermined values of I and B:

$$\tau_{net} = \left[\frac{B_o ab[\omega_B \sin(\omega_B t)\sin(\omega_C t) - \omega_C \cos(\omega_C t)\cos(\omega_B t)]}{R}\right] ab\left[B_o \cos(\omega_B t)\right]\cos(\omega_C t)$$

By simplifying the equation we obtain a general expression for the torque as a function of time:

$$\tau(t) = \frac{B_o^2 a^2 b^2}{R}\left[\omega_B \sin(\omega_B t)\sin(\omega_C t) - \omega_C \cos(\omega_C t)\cos(\omega_B t)\right]\cos(\omega_B t)\cos(\omega_C t) \tag{7}$$

Nonetheless, given the extensive trigonometric terms in our function, a question presents itself. To what extent can we properly analyse the behaviour of the torque?

3.1.3 Evaluation

To study the behaviour of the torque with respect to time, we have to assign values to each constant as to obtain graphs that can be properly analyzed. According to equation 7, the theoretical behaviour depends on two factors; the magnitudes outside of the brackets and the angular velocities. The magnitudes in $\frac{B_o^2 a^2 b^2}{R}$ have no greater effect on the behaviour, other than influencing the amplitude of the possible oscillation. However, the magnitude of each angular velocity (ω_B and ω_C) has a direct effect on the behaviour of the torque as these are embedded in the trigonometric terms of the function.

If we plot our expression assuming $\frac{B_o^2 a^2 b^2}{R} = 1$ and assigning different values to each angular velocity, we observe there are many possible behaviours for the torque.

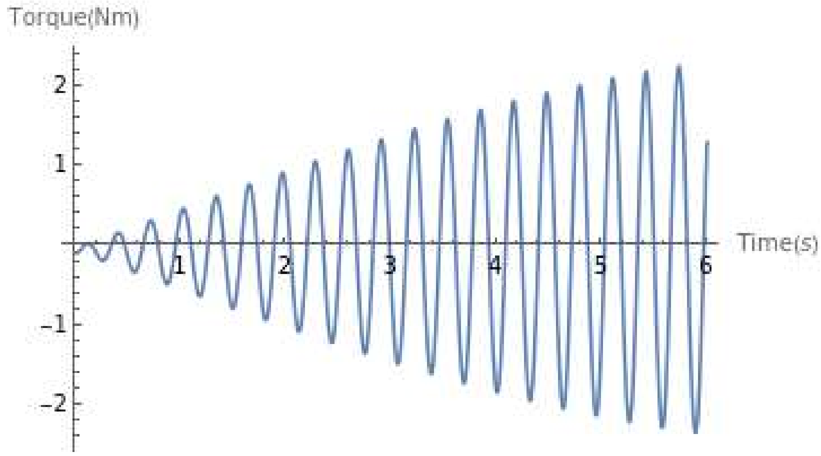

Figure 7: Torque vs time graph. When $\omega_B = 10\frac{rad}{s}$ and $\omega_C = 0.1\frac{rad}{s}$. Own Image. Made using Wolfram Mathematica [8]

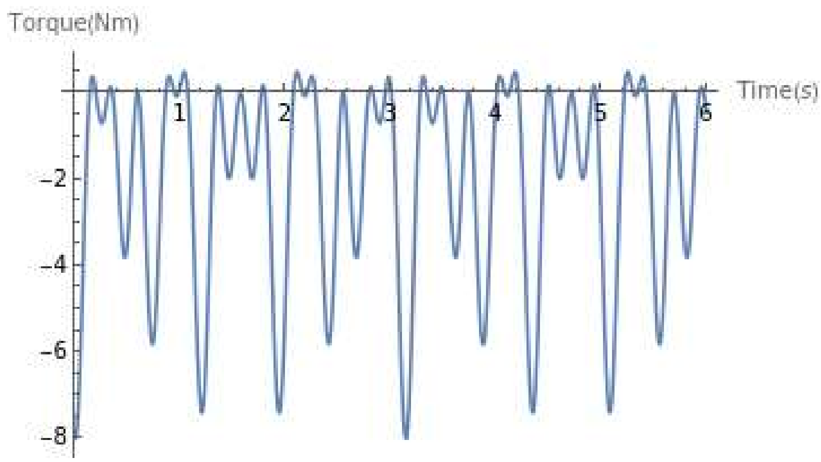

Figure 8: Torque vs time graph. When $\omega_B = 5\frac{rad}{s}$ and $\omega_C = 8\frac{rad}{s}$. Own Image. Made using Wolfram Mathematica [8]

For that reason, to narrow down the scope of our investigation and limit the complexity of the behaviour we will analyze two cases where we reduce the angular velocity to only one.

3.2 Particular Cases

3.2.1 Case 1: Stationary Magnetic Field and Rotating Loop

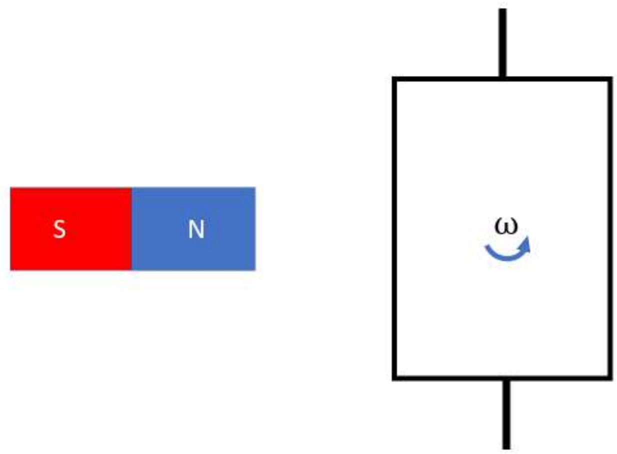

Figure 9: Magnet is stationary and loop is rotating

Similar to an induction generator, in this first case, the magnet remains stationary and only the rectangular loop will rotate. To reach an expression for the behaviour of the torque in this scenario, we will start from the general equation (7). If the magnetic field is stationary, then its angular velocity is equal to zero ($\omega_B = 0$). Therefore, we can express the function of torque as:

$$\tau(t) = \frac{B_o^2 a^2 b^2}{R}\left[0\sin(0t)\sin(\omega_C t) - \omega_C\cos(\omega_C t)\cos(0t)\right]\cos(0t)\cos(\omega_C t)$$

$$\tau(t) = \frac{B_o^2 a^2 b^2}{R}\left[-\omega_C\cos(\omega_C t)\right]\cos(\omega_C t)$$

We can omit the specification of the angular velocity as there is only one rotating object in the system . Thus, when only the loop rotates, the behaviour of torque with respect to time is depicted by the following function:

$$\tau(t) = -\frac{B_o^2 a^2 b^2}{R}\omega\cos^2(\omega t) \tag{8}$$

3.2.2 Case 2: Magnetic Field and Loop rotate at the same angular velocity

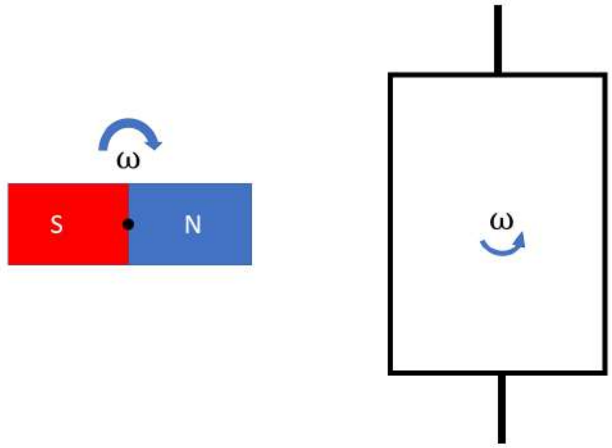

Figure 10: Magnet and loop rotate at the same angular velocity

For the second case the magnet and the rectangular loop will rotate, like in an induction motor. But unalike an induction motor they will rotate in sync. Therefore, we can simplify the general equation considering their angular velocities are the same ($\omega_B = \omega_C = \omega$).

$$\tau(t) = \frac{B_o^2 a^2 b^2}{R} \left[\omega \sin(\omega t) \sin(\omega t) - \omega \cos(\omega t) \cos(\omega t)\right] \cos(\omega t) \cos(\omega t)$$

$$\tau(t) = \frac{B_o^2 a^2 b^2}{R} \omega \left[\sin(\omega t) \sin(\omega t) - \cos(\omega t) \cos(\omega t)\right] \cos^2(\omega t)$$

To simplify the equation inside the brackets we apply the trigonometric property: $\sin^2(x) - \cos^2(x) = -\cos 2x$. This way reaching a desired function for the torque:

$$\tau(t) = -\frac{B_o^2 a^2 b^2}{R} \omega \cos(2\omega t) \cos^2(\omega t) \tag{9}$$

4 Experimentation

4.1 Data Recollection

Using an induced current simulator, the torque over the loop at different instants in time was obtained. For those same instants, we will calculate the torque using our equations. This will be done with the purpose of comparing how the simulated experimental data compares with the theoretical model.

However, we must first assign values for the initial conditions of the simulated system. For the dimensions of the rectangular loop, we assigned relatively realistic values, assuming a rectangular loop of wire is typically not tremendous. For the intensity of the magnetic field and the resistance of the loop, we assigned values that would result in an amplitude that is not too significant. Furthermore, the angular speed for both cases will be a sensible value, as shown in the table below.

Variable	Magnitudes
B_o	10.0 T
a	0.40 m
b	0.20 m
R	3.00 Ω
ω	5.00 $rads^{-1}$

Table 1: Initial conditions for simulator

4.1.1 Case 1: Stationary Magnetic Field and Rotating Loop

For case 1, the magnetic field is stationary and the loop rotates at 5 $rad.s^{-1}$. The simulator gives the following values for the torque at the following instants:

Time (s)	Torque (Nm)
0.12	-0.721
0.24	-0.123
0.37	-0.081
0.51	-0.742
0.64	-1.065
0.74	-0.758
0.88	-0.089
1.01	-0.115
1.13	-0.716
1.26	-1.066

Table 2: Case 1 - Experimental values

To calculate the theoretical values for the torque at the same instants as the simulator we will employ equation 8:

$$\tau(0.12) = -\frac{(10T)^2(0.4m)^2(0.2m)^2}{3\Omega}\left(5rads^{-1}\right)\cos^2\left(\left(5rads^{-1}\right)(0.12s)\right)$$

$$\tau(0.12) = -0.727Nm$$

We will repeat this process for every instant of time in the data obtained by the simulator. In this manner, formulating a table that presents our theoretical values:

Time (s)	Torque (Nm)
0.12	-0.727
0.24	-0.140
0.37	-0.081
0.51	-0.735
0.64	-1.063
0.74	-0.767
0.88	-0.101
1.01	-0.117
1.13	-0.693
1.26	-1.066

Table 3: Case 1 - Theoretical values

4.1.2 Case 2: Magnetic Field and Loop rotate at the same angular velocity

For case 2, both the magnet and the loop rotate at 5 $rad.s^{-1}$. The simulator gives the following values for the torque at the following instants:

Time (s)	Torque (Nm)
0.14	-0.136
0.26	0.063
0.39	0.100
0.49	-0.147
0.64	-1.063
0.76	-0.163
0.87	0.105
0.99	0.053
1.11	-0.068
1.26	-1.067

Table 4: Case 2 - Experimental values

Employing equation 9 to calculate the theoretical values of torque at the same instants as the simulator:

$$\tau(0.14) = -\frac{(10T)^2(0.4m)^2(0.2m)^2}{3\Omega}(5rads^{-1})\cos(2(5rads^{-1})(0.14s))\cos^2((5rads^{-1})(0.14s))$$

$$\tau(0.14) = -0.106Nm$$

Repeating this process for every instant of time in the data obtained by the simulator. In this manner, formulating a table that presents our theoretical values:

Time (s)	Torque (Nm)
0.14	-0.106
0.26	0.065
0.39	0.106
0.49	-0.118
0.64	-1.056
0.76	-0.168
0.87	0.100
0.99	0.053
1.11	-0.061
1.26	-1.066

Table 5: Case 2 - Theoretical values

5 Data Analysis

5.1 Approach

With the data we have recollected, using Excel, we will plot the points in a torque vs time graph. Considering we have a function for the torque, in each case, we can extend the tables of theoretical values for infinitesimal instances, generating a proper plot of the theoretical model. Thereupon, we will situate the points extracted from the simulator in a graph with the theoretical values, observing how the experimental data adjusts to the theoretical model. Furthermore, we will compare both cases and expand on our work by determining a relationship between the maximum torque and the angular velocity.

5.2 Case 1: Stationary Magnetic Field and Rotating Loop

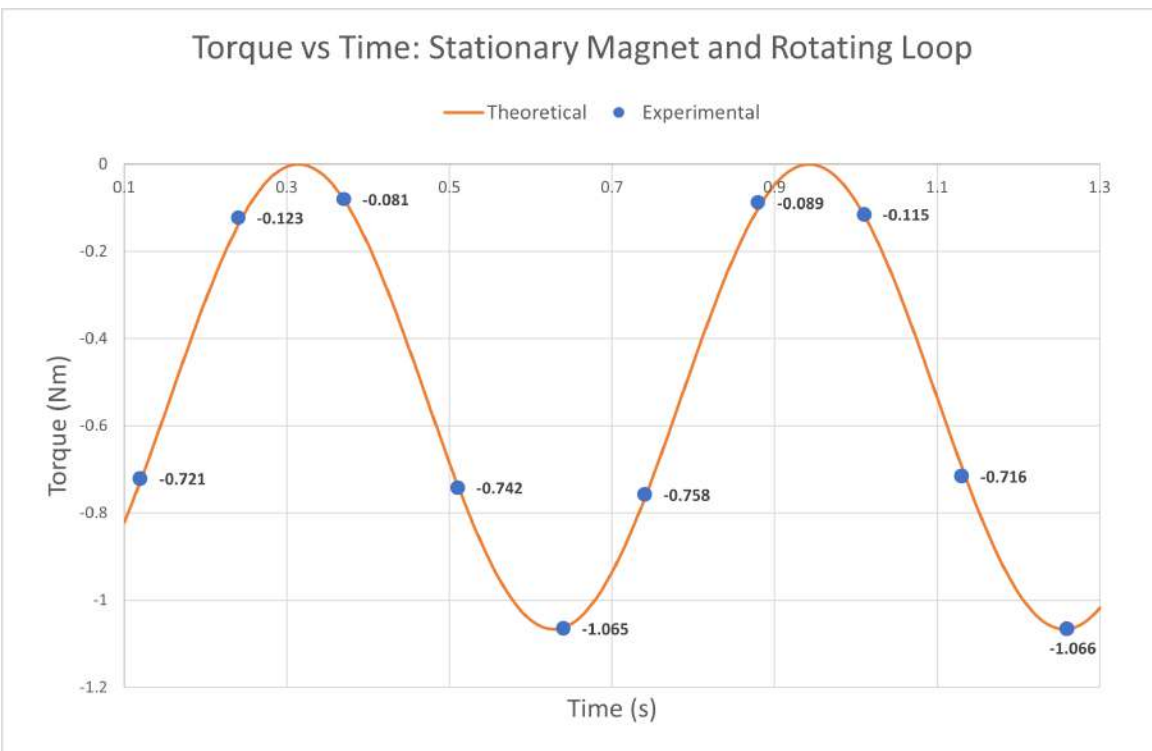

Figure 11: Comparing the theoretical and experimental data of Case 1. Made in Excel.

Observing figure 11, when only the loop rotates and the magnet is stationary, the theoretical model for the torque with respect to time, shows an oscillating behaviour. The torque oscillation takes place exclusively underneath the time axis, indicating a torque that only has negative values. Furthermore, we observe how the values of the torque obtained by the simulator align almost to perfection with our theoretical model. We can say that our results were as expected.

5.3 Case 2: Magnetic Field and Loop rotate at the same angular velocity

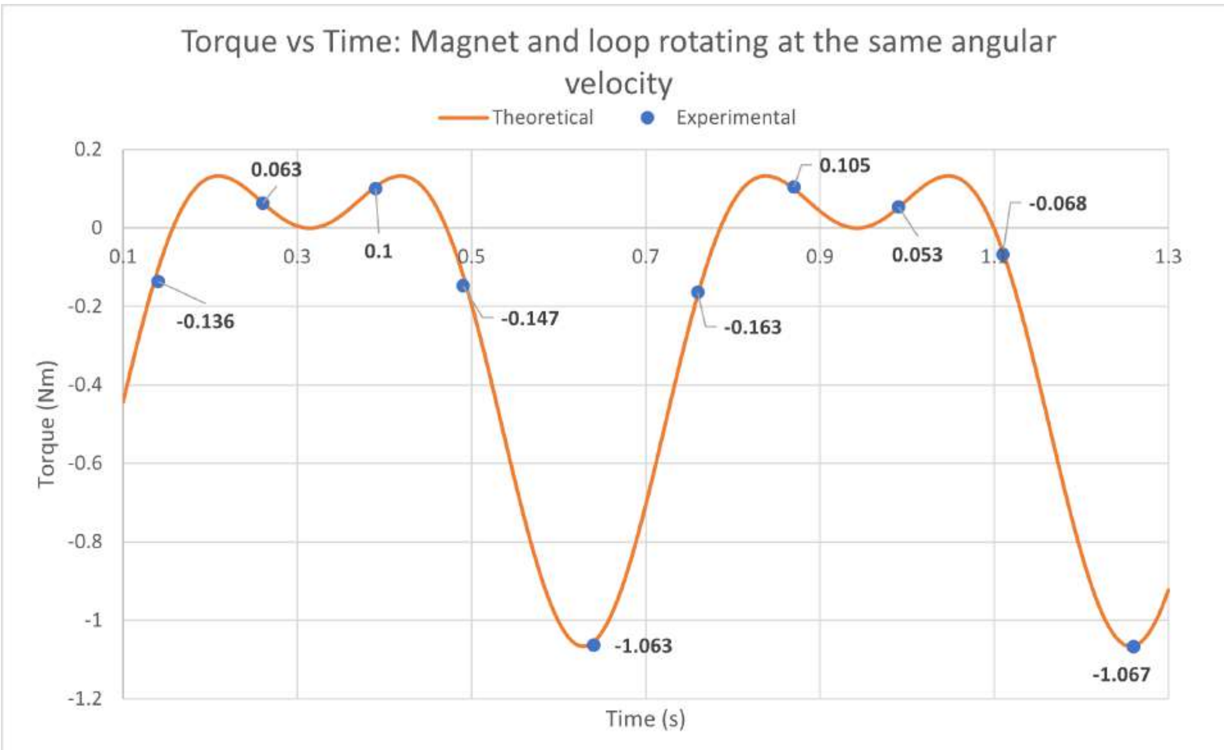

Figure 12: Comparing the theoretical and experimental data of Case 2. Made in Excel.

Observing figure 12, when both the magnet and loop rotate at the same velocity, the theoretical model for the torque with respect to time, shows an irregular oscillating behaviour. Where the oscillation has 2 crests and three troughs. Also, it is pertinent that we point out the torque oscillates temporarily above the time axis, the implications of this will be discussed in the conclusions. Furthermore, we observe how the data generated by the simulator coincides to great degree with the theoretical behaviour of the torque. We can say our results where as expected.

5.4 Comparing both Cases

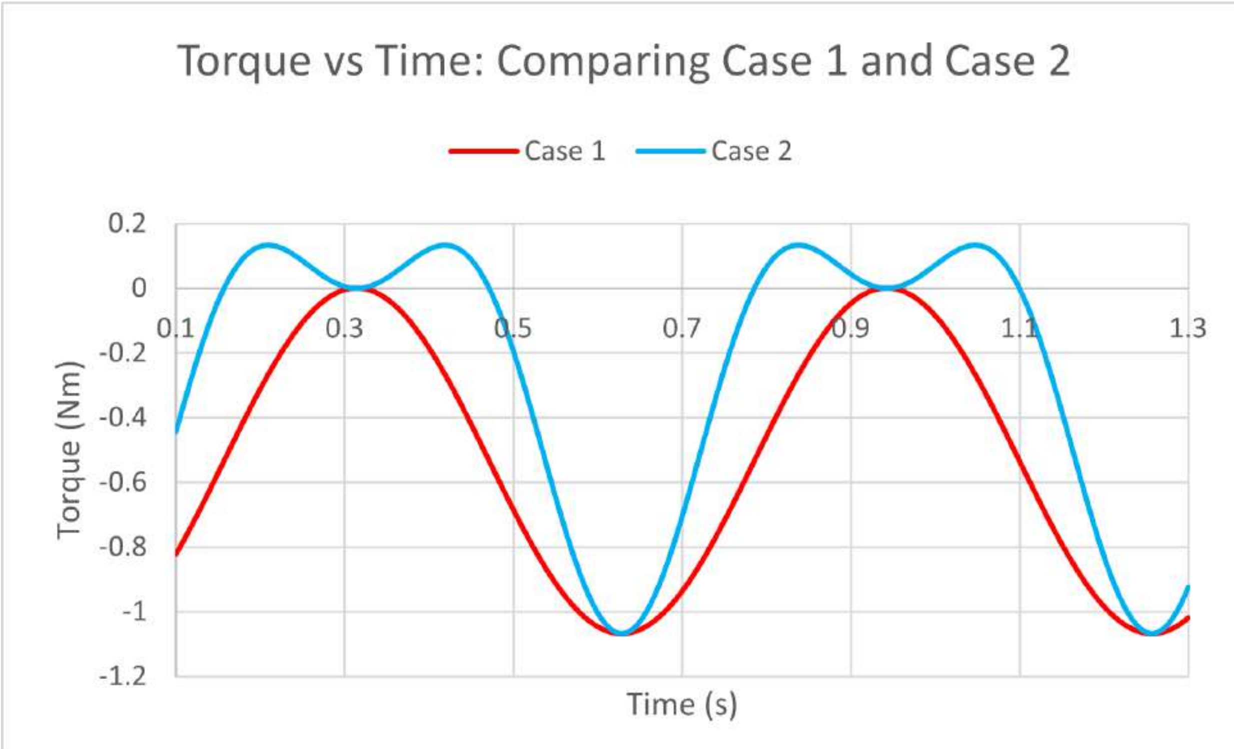

Figure 13: Comparing the plots of case 1 and case 2. Made in Excel.

We can plot and compare both oscillations according to their theoretical behaviour as there is little margin of error with the experimental data. Observing figure 13 the crest of Case 1's oscillation coincides with the trough of the concavity of Case 2. Secondly, we observe how the maximum magnitude of torque is the same for both and occurs at the same instant. Lastly, the torque is zero far more frequently when both magnet and loop rotate than when only the loop rotates. Nonetheless, the most noticeable difference with both plots is that Case 2 has values for positive torques while Case 1 does not.

5.5 Relationship between Maximum Torque and Angular Velocity

Now that we have seen how torque behaves with respect to time, we are interested to extend this investigation and see how the magnitude of maximum torque varies according to different angular velocities.

As we observed in figure 13, for both cases the magnitude of the maximum torque is the same and occur at the same instants. Likewise, observing equation 8 and equation 9 when $t = 0$ there will be a maximum value for the torque. This is only considering the loop or both loop and magnet are initially rotating. Thus, theoretically, the relation between the maximum torque and angular velocity should obey the following equation:

$$\tau_{max} = -\frac{B_o^2 a^2 b^2}{R}\omega$$

Using the simulator, we obtained values for the maximum torque at different angular velocities. We plot this data using Excel and using Excel's data analysis tool for the linear regression.

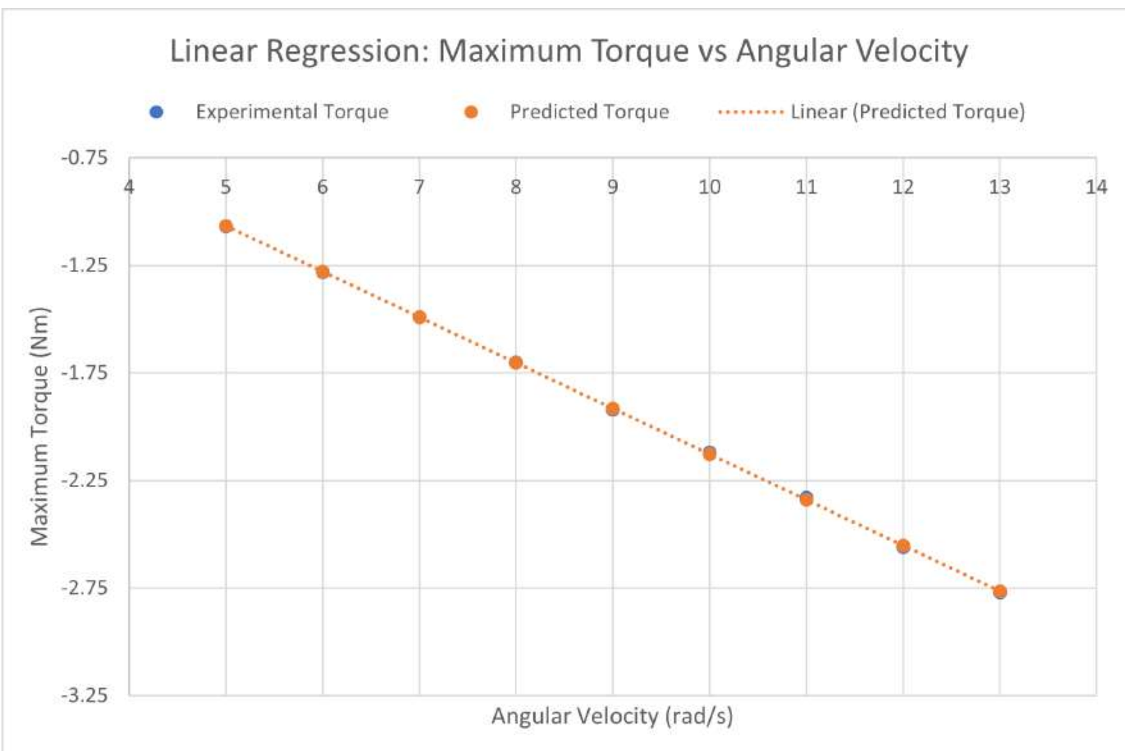

Figure 14: Linear regression of Maximum Torque vs Angular Velocity. Made in Excel.

We observe that the angular velocity and the negative magnitude of torque are directly proportional. Moreover, according to Excel, the slope of the linear regression function with its respective uncertainty is:

$$-0.2123 Nms \pm 0.0084 Nms$$

Where the uncertainty is minimal because the experimental data points lie almost completely under the linear regression function. If we compare this to our theoretical slope:

$$-\frac{B_o^2 a^2 b^2}{R} = -\frac{(10T)^2 (0.4m)^2 (0.2m)^2}{(3\Omega)^2} = -0.2133 Nms$$

We validate the certainty of our theoretical model as both slopes are essentially the same. This premise is reinforced by calculating the percentile error:

$$\% Error = \frac{\mid 0.2133 - 0.2123 \mid}{0.2133} 100\% = 0.469\%$$

6 Conclusions and recommendations

According to equation 8, when the magnetic field is stationary we expected an oscillatory behaviour for the torque because of the identical cosine terms ($\cos^2 x$) presented in the function. On the other hand, according to equation 9, when the magnet rotates at the same velocity as the loop, we expected a more complex oscillatory behaviour for the torque because the function includes two different cosine terms [$\cos(2\omega t)\cos^2(\omega t)$]. Plotting both functions we conclude that the torque behaves as expected. As we can observe in figure 11 and figure 12 the experimental data from the simulator aligns almost perfectly with the theoretical behaviour. Error bars were obtained but were insignificant compared to the scale of the graph. This suggests that our theoretical work has been correct.

However, the question still remains, what do these behaviors of the torque implicate on the rotation of the loop? According to figure 13 when the magnet is stationary the torque will always have a negative magnitude, thus showing a torque that will always oppose the rotation of the loop. However, if the magnet and loop rotate, the torque can assume positive magnitudes. Meaning it will spend most time opposing the rotation but for short periods of time, the magnetic torque will have the same direction as the loop's rotation. Presumably, this would suggest a temporary increase in the velocity of the loop, as the torque should complement its rotation. Therefore, we can conclude, that for our system to be functional, the loop must experience another torque of the same magnitude but in the opposite direction as the magnetic torque, to cancel out its effects.

The relation between the maximum magnitude of the magnetic torque and the angular velocity was explored in figure 14. Where we concluded, the faster the magnetic field or both field and loop rotate, the greater will be the magnetic torque generated that opposes the rotation of the loop. This relationship coincides with the theoretical presumption. As according to Faraday's law (3), the faster the magnetic flux over the loop varies with respect to time, the greater the current induced on the loop, consequently, greater torque should be generated. Likewise, the slope for the experimental linear regression according to Excel is almost identical to our theoretical slope, with an insignificant percentile error of 0.47%. In this manner, we can conclude, that the system behaves accordingly to our theoretical analysis. Marking the success of the theoretical analysis.

Our investigation was based on an ideal system, where the loop can rotate at a constant angular velocity despite experiencing an opposing magnetic torque. However, since we were only interested on the behaviour of the magnitude of the magnetic torque and its hypothetical effect on the loop, there was no harm done omitting such factors. Like this one, there are many others that were not considered in our system. For example; eddy currents, air friction, magnetic dipoles, among others. Apart from the forementioned, it would be interesting to add new components and conditions to the system: having more than one magnet or considering a 3 dimensional loop. However, what could be the most interesting way of expanding this investigation, is considering the torque does affect the rotation of the loop, so its angular velocity will no longer be constant.

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A R.M.F of an AC Induction Motor

The rotating magnetic field in modern AC induction motors is produced from 3 pairs of electromagnets[2]. The conductors are separated by 120 degrees and the currents are each out of sync by 120 degrees. Having 3 alternating currents, we have 3 alternating magnetic fields. As the fields are a direct product of the current we attribute them the same sinusoidal behaviour. Therefore we can write the flux of each field as the following:

$$\phi_A = \phi_{Max} \sin(\omega t)$$

$$\phi_B = \phi_{Max} \sin(\omega t + \frac{2\pi}{3})$$

$$\phi_C = \phi_{Max} \sin(\omega t - \frac{2\pi}{3})$$

The magnetic fields interact to form a resulting field. Each field changes in direction and is present only for certain intervals of time. The flux of the fields will interact in specific points, their interactions over time simulate a single rotating field.

[2] In simple terms: currents passing through a conductor

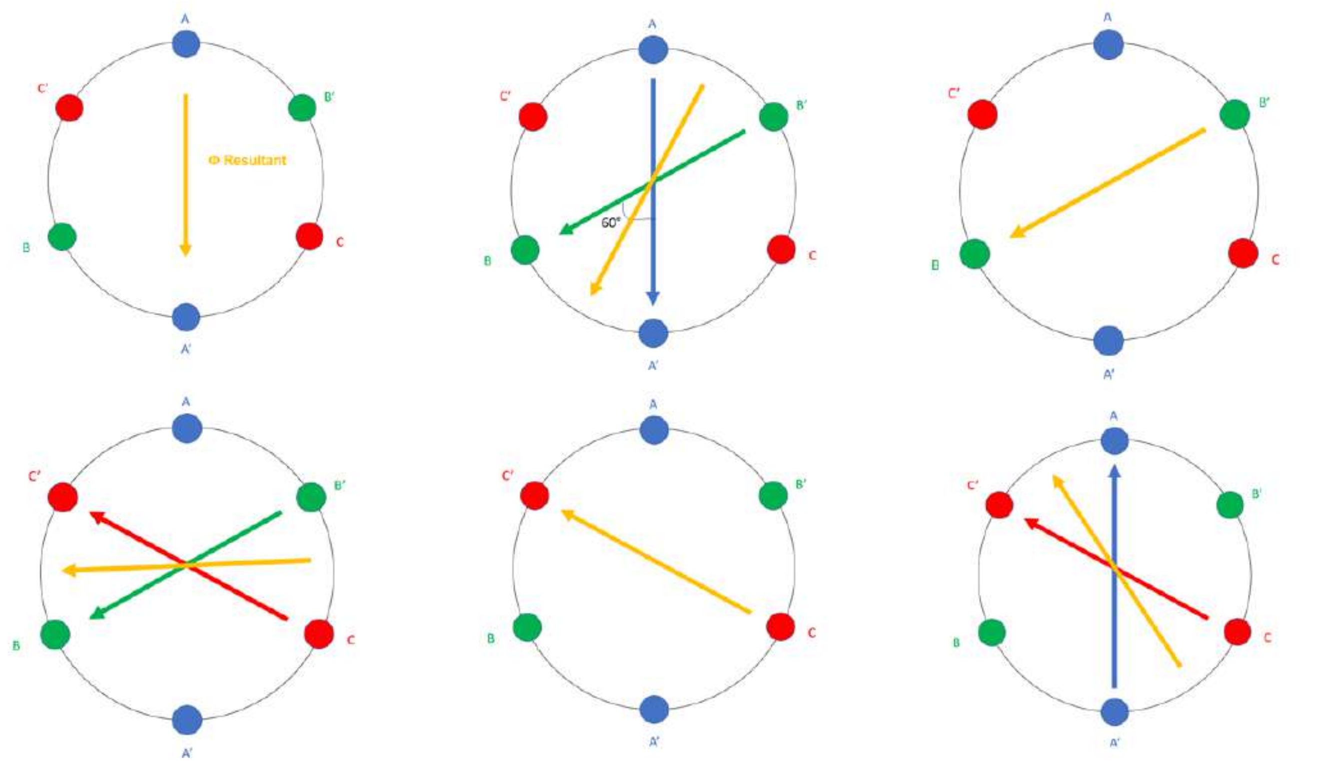

Figure 15: Varying flux in RMF. Own Image